Charged and Neutral Current Interactions

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Introduction

So far, we have been introduced to topics in the standard model in a slightly non-linear way, covering gauge theory, the Higgs mechanism, and generation of fermion masses through their interaction with the Higgs boson. In this report, we will derive the form of the charge and neutral current interactions. We have already covered part of the neutral sector with the derivation of the coupling of the photon. We will cover the rest here.

Leptons

Charged interactions

We have already constructed a gauge-covariant derivative

$$D_\mu = \partial_\mu + igW^a_\mu T^a + ig' Y B_\mu.$$

We can write our Lagrangian as

$$i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + i\overline{e_R}\gamma^\mu D_\mu e_R.$$

Here, ψ_L is the lepton doublet

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix},$$

while e_R is a lepton singlet. We do not include ν_R because there has been no evidence of right-handed neutrinos. The left- and right-handed fields are obtained by having the projection operators P_L and P_+ on the lepton field. The $SU(2)$ transformation only acts on the left-handed field, while the $U(1)$ transformation acts on both the left- and right-handed fields. We will first derive the interactions for the left-handed field, and then consider the right-handed field. The fermion-fermion interaction term in the Lagrangian ($i\overline{\psi_L}\not{D}\psi_L$) is, explicitly:

$$-\frac{g}{2}\begin{pmatrix} \overline{\nu_L} & \overline{e_L} \end{pmatrix}\gamma^\mu \begin{pmatrix} W^3_\mu & W_1 - iW_2 \\ W_1 + iW_2 & -W^3_\mu \end{pmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - g'\left(\frac{1}{2}\right)\begin{pmatrix} \overline{\nu_L} & \overline{e_L} \end{pmatrix}\gamma^\mu B_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

We then identify the combinations of W_1 and W_2 as our actual $W^\pm_\mu$ bosons. If we consider just the term on the left, expand it out, and isolate the charge current interactions, that is, the terms with $W_\mu^\pm$ we get

$$\frac{-g}{\sqrt{2}}\overline{\nu_L}\gamma^\mu W_\mu^+ e_L - \frac{g}{\sqrt{2}}\overline{e_L}\gamma^\mu W_\mu^- \nu_{e_L}$$

We can piece together two such diagrams after generalizing to other generations of leptons to give a plausible description of the decay mechanism of the muon.

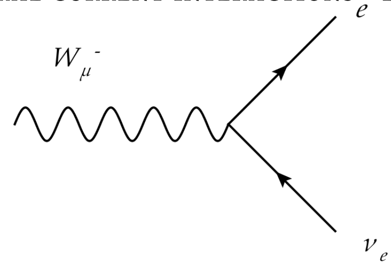

Figure 2: Feynman diagram for negative charge current interaction.

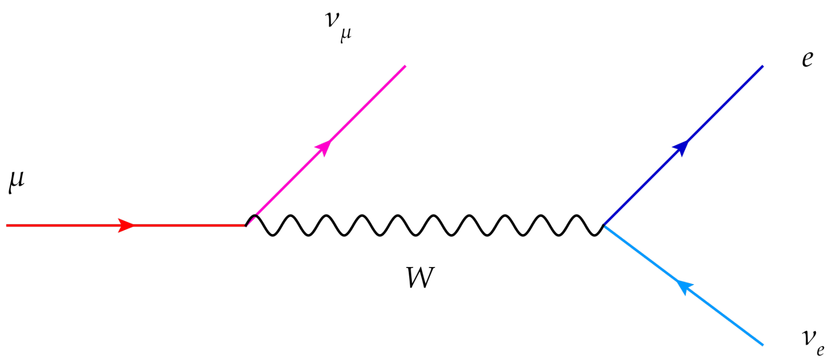

Figure 3: Feynman diagram for muon decay through a weak charge interaction.

Neutral interactions

We have already covered how the photon field couples to charged fields. Now we will examine the other kind of neutral interaction, that of the Z boson. Similar to how we picked out the terms with the charged W Boson interactions previously, this time we will pick out the terms with the W_μ^3 and the B_μ fields. Doing so, we get the following terms.

$$-\frac{g}{2}\overline{\nu_L}\gamma^\mu W_3 \nu_L - g' Y \overline{\nu_L}\gamma^\mu B_\mu \nu_L + \frac{g}{2}\overline{e_L}\gamma^\mu W_\mu^3 e_L - g' Y \overline{e_L}\gamma^\mu B_\mu e_L$$

If we write out W_μ^3 and B_μ in terms of our force carrier fields Z_μ and A_μ, we get[1]

$$-(gT_3\cos\theta - g'Y\sin\theta)\overline{\nu_L}\gamma^\mu Z_\mu \nu_L - (gT_3\sin\theta + g'Y\cos\theta)\overline{\nu_L}\gamma^\mu A_\mu \nu_L$$

If we look at the Z_μ coupling, we get (after factoring out the '-' sign and redefining Y as $Q - T_3$)[2]

$$\begin{aligned}(gT_3\cos\theta - g'Y\sin\theta) &= \frac{e\cos\theta T_3}{\sin\theta} - \frac{e\sin\theta}{\cos\theta}(Q - T_3)\\ &= \frac{e}{\sin\theta\cos\theta}(T_3 - Q\sin^2\theta)\end{aligned}$$

[1] Here we write the terms in front of both the third component of the weak isospin, T_3 and the hypercharge Y (Earlier we had explicitly applied the isospin operator to the the fermion fields).

[2] We substitute the values of the coupling constants g and g' in terms of e and θ as well.

We redefine the combination of operators $(T_3 - Q\sin^2\theta)$in the line above as the coupling constant for Z boson interactions, g_Z^L. This gives us the basic coupling term for the Z boson and leptons,

$$\frac{e}{\sin\theta\cos\theta} g_Z^L \overline{l_L} \gamma^\mu Z_\mu l_L$$

For the right-handed lepton l_R, we have the Z_μ coupling as:

$$\frac{e}{\sin\theta\cos\theta} g_Z^R \overline{e_R} \gamma^\mu Z_\mu e_R$$

For the right-handed electron, $g_Z^R = \sin^2\theta$ and for the right-handed neutrino, $g_Z^R = 0$ because of their particular values of the third component of weak hypercharge and weak isospin.

Quarks

The interaction terms for the quarks arise from the kinetic term of the Lagrangian. We write the Lagrangian in terms of the left-handed quark doublet and right-handed quark singlet fields,

$$Q_{L_i} = \begin{pmatrix} U_{L_i} \\ D_{L_i} \end{pmatrix}; \qquad U_{R_i} \text{ and } D_{R_i}.$$

Note that this time, we have a doublet right-handed field, because we know that right-handed quarks that correspond to the left-handed quarks do exist. Thus, the kinetic term of the Lagrangian becomes[3]

$$\mathcal{L}_{\text{KE}} = \sum_i i\overline{Q_{L_i}}(\partial_\mu + igW_\mu^a T^a + ig'B_\mu Y)Q_{L_i} + i\overline{U_{R_i}}(\partial_\mu + ig'B_\mu Y)U_{R_i} + i\overline{D_{R_i}}(\partial_\mu + ig'B_\mu Y)D_{R_i}$$

Isolating the interaction terms, we get[4]

$$-\frac{g}{\sqrt{2}}\begin{pmatrix}\overline{U_{L_i}} & \overline{D_{L_i}}\end{pmatrix}\begin{pmatrix}\frac{W_\mu^3}{\sqrt{2}} & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}U_{L_i} \\ D_{L_i}\end{pmatrix} - g'\begin{pmatrix}\overline{U_{L_i}} & \overline{D_{L_i}}\end{pmatrix}\begin{pmatrix}\frac{B_\mu}{6} & \\ & \frac{B_\mu}{6}\end{pmatrix}\begin{pmatrix}U_{L_i} \\ D_{L_i}\end{pmatrix} - g'\begin{pmatrix}\overline{U_{R_i}} & \overline{D_{R_i}}\end{pmatrix}\begin{pmatrix}\frac{2B_\mu}{3} & \\ & -\frac{B_\mu}{3}\end{pmatrix}\begin{pmatrix}U_{R_i} \\ D_{R_i}\end{pmatrix}$$

Charged Interactions

The charged interaction terms are

$$-\frac{g}{\sqrt{2}}\left[\overline{U_{L_i}}W_\mu^+ D_{L_i} + \overline{D_{L_i}}W_\mu^- U_{L_i}\right]$$

Thus, we see that there are flavor changing charge currents at the tree level itself.

[3] Where the index i denotes the three generations of quarks.

[4]

$$YQ_{L_i} = \begin{pmatrix}\frac{1}{6} & \\ & \frac{1}{6}\end{pmatrix}Q_{L_i}$$

$$YQ_{R_i} = \begin{pmatrix}\frac{2}{3} & \\ & -\frac{1}{3}\end{pmatrix}Q_{R_i}$$

Neutral Interactions

Collecting the neutral interaction terms, we get

$$-\frac{g}{2}\left(\overline{U_{L_i}}W_\mu^3 U_{L_i} + \overline{D_{L_i}}W_\mu^3 D_{L_i}\right) - g'\left(\overline{U_{L_i}}\frac{B_\mu}{6}U_{L_i} + \overline{D_{L_i}}\frac{B_\mu}{6}D_{L_i} + \overline{U_{R_i}}\frac{2B_\mu}{3}U_{R_i} + \overline{D_{R_i}}\frac{(-B_\mu)}{3}D_{R_i}\right)$$

Combining terms, we get the expression

$$\overline{U_{L_I}}\left(-\frac{g}{2}W_\mu^3 - \frac{g'}{6}B_\mu\right)U_{L_i} + \overline{D_{L_i}}\left(-\frac{g}{2}W_\mu^3 - \frac{g'}{6}B_\mu\right)D_{L_i} + \overline{U_{R_i}}\left(-\frac{2g'}{3}B_\mu\right)U_{r_i} + \overline{D_{R_i}}\left(+\frac{g'}{3}B_\mu\right)D_{R_i}$$

Now, we know from earlier that

$$\begin{aligned} g &= \frac{e}{\sin\theta} \\ g' &= \frac{e}{\cos\theta} \\ W_\mu^3 &= \cos\theta Z_\mu + \sin\theta A_\mu \\ B_\mu &= -\sin\theta Z_\mu + \cos\theta A_\mu \end{aligned}$$

If we substitute these into the interaction terms above, we get the neutral currents in the *gauge eigenstate basis*.

Z-boson current

$$-\frac{e}{2}\left(\cot\theta - \frac{\tan\theta}{3}\right)\left(\overline{U_{L_i}}Z_\mu U_{L_i} + \overline{D_{L_i}}Z_\mu D_{L_i}\right) - \frac{2e}{3}\tan\theta\left(\overline{U_{R_i}}Z_\mu U_{R_i}\right) + \frac{e}{3}\tan\theta\left(\overline{D_{R_i}}Z_\mu D_{R_i}\right)$$

Photon current

$$-\frac{2e}{3}\left(\overline{U_{L_i}}A_\mu U_{L_i} + \overline{D_{L_i}}A_\mu D_{L_i}\right) + \frac{2e}{3}\left(\overline{U_{R_i}}A_\mu U_{R_i}\right) - \frac{e}{3}\left(\overline{D_{R_i}}A_\mu D_{R_i}\right)$$

It is not necessary that the gauge/flavor eigenstates are the same as the mass eigenstates. In fact, the mass eigenstates are linear combinations of the gauge eigenstates.

We can also write the charged interactions in terms of the mass eigenstates by employing the Kobayashi-Maskawa flavor mixing matrix.

$$V_{KM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The charged current induces flavor mixing, determined by V_{KM}.

Questions

1. Relationship between mass and gauge eigenstates, and how to convert between them?