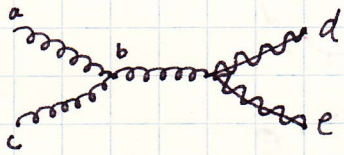
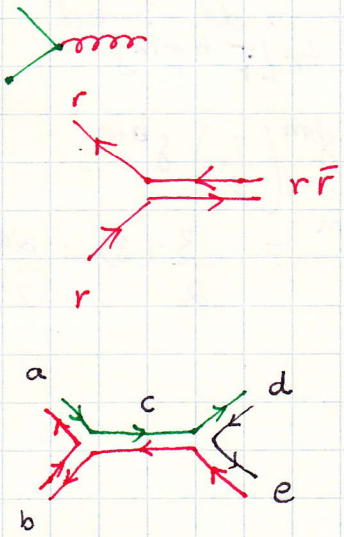
Group factors



When we take $|M|^2$, we get a trace term ~~multiplying~~ as a multiplicative factor

$$f^{abc}\, f^{dbe}$$

Color conservation @ at quark-gluon vertex.



When we take $M^* M$, we get

~~f^{abc}~~ ~~f^{abc}~~ ~~f^{dbe}~~

$$f^{abc}\, f^{dbe}\, f^{amc}\, f^{dme}$$

Moving factors around,

$$f^{abc}\, f^{amc}\, f^{dbe}\, f^{dme}$$

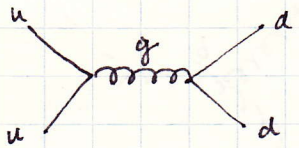
$$= f^{bac}\, f^{mac}\, f^{bde}\, f^{mde}$$

$$= C_2(G)\,\delta^{bm}\, C_2(G)\,\delta^{bm}$$

$$= 3 \cdot 3 \cdot \delta^{bm}\delta^{bm}$$

$$= 9 \times 8 = \boxed{72}$$

From Peskin,



The QCD ~~factor~~ diagram contains the factor

$$(t^a)_{i'i}\,(t^a)_{j'j}$$

i, i' are the initial & final colours of the u quark & j, j' are the initial & final colors of the d quark.

Squaring this, $t^a_{i'i}\, t^a_{j'j}\, t^b_{i'i}\, t^b_{j'j}$

$$= (-t^a_{i'i}\, t^b_{ii'})(-t^b_{j'j}\, t^b_{jj'})$$

$$= \mathrm{tr}[t^a t^b]\,\mathrm{tr}[t^a t^b]\,\delta^{ab}\delta^{ab}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot 8$$

Averaging over initial ~~spins~~ colors: $\frac{1}{3} \times \frac{1}{3}$ @

$\Rightarrow$ ~~total factor~~ $= \frac{1}{36}$

Sum over final colors: @

$\Rightarrow$ total factor $= \frac{8}{36} = \boxed{\frac{2}{9}}$

$\delta^{ab}\delta^{ab}$ ~~$\delta^{11}\delta^{11} + \delta^{12}\delta^{12} + \delta^{13}\delta^{13}$~~ .

$= \delta^{11}\delta^{11} + \delta^{12}\delta^{12} + \ldots$

$+ \delta^{22}\delta^{22}$

$+ \delta^{33}\delta^{33}$

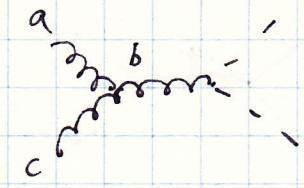
$+ \ldots$

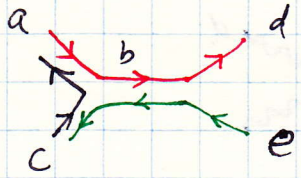
8 gluon colors

$\Rightarrow$ factor of 8.

@

Group factors cont'd.





~~$\mathcal{M}^* \mathcal{M}$~~

$$f^{abc} T^b_{de}$$

$\mathcal{M}^* \mathcal{M}$ gives us: $f^{abc} T^{b*}_{de} \cdot f^{amc} T^{m}_{de}$

$$= f^{abc} f^{amc} T^{b*}_{de} T^{m}_{de}$$

$$= f^{bac} f^{mac} T^{b*}_{de} T^{m}_{ed}$$

$$= C_2(G)\,\delta^{bm}\, \mathrm{tr}[T^b T^m]$$

$$= (3)\,\delta^{bm}\left(\frac{1}{2}\right)\delta^{bm}$$

$$= \frac{3}{2}\,\delta^{bm}\delta^{bm} = \frac{3}{2}(8) = \frac{24}{2} = \boxed{12}$$

If there was a chance of a – sign from the c.c. ~~the~~ cancels it