Conservation of Angular Momentum

1. Demos

(i) Person on spinning chair with weights
Bring weights in → speed up ~~(KI)~~
Bring weights out → slow down.

Conservation of angular momentum : $I\omega = I'\omega'$

$\Rightarrow L_i = L_f$

$K_i = \frac{1}{2} I\omega^2, \quad K_f = \frac{1}{2} I'\omega'^2 = \frac{1}{2} I' \frac{I^2\omega^2}{I'^2}$

$\Rightarrow \frac{K_f}{K_i} = \frac{I}{I'}$. If $I < I'$, $K_f < K_i$

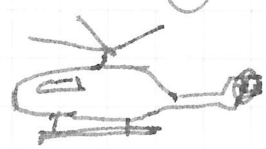
What happens to the energy?
You are doing work ⇒ there is a change in kinetic energy (Weight want to fly out, you oppose that motion ⇒ negative work done on the system.)

(ii) Person on chair with bicycle wheel.
(a) ~~State~~ start with wheel counterclockwise, flip 180° ⇒ chair (+ person) spins ccw.
(b) Start w/ horizontal axis, flip to vertical.

What spun the chair? friction between the chair and person.
What spun the person? Normal force on person from axle.

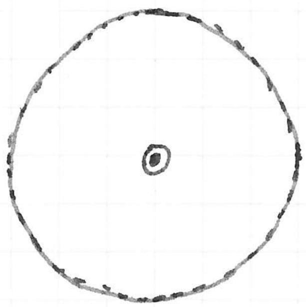
(iii) Gyroscope.

Also, helicopters → stability propeller.

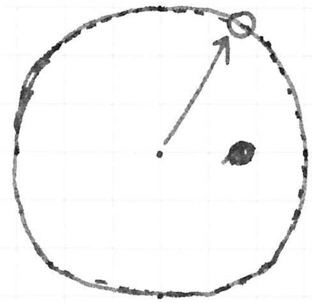


Example: Child on a merry-go-round.

Q: A 36-kg child stands at the center of a merry-go-round of mass 87 kg and radius 2.2 m spinning at 1 rev/s. Find the angular velocity of the system after the child walks to the edge.



initial



final

No external torques ⇒ Angular momentum of the system is conserved.

$$\Rightarrow L_i = L_f$$

$$\Rightarrow I_i \omega_i = I_f \omega_f$$

I_i = initial moment of inertia of the system

~~= moment of inertia of disc + that I of the~~

$$= I_{disc(i)} + I_{child(i)}$$

$$= \frac{MR^2}{2} + m r_i^2$$

[M = mass of disc, R = radius of disc, m = mass of child] [r = distance of child from axis]

$$= \frac{MR^2}{2} + m(0)^2$$

$$= \frac{MR^2}{2}$$

$$I_f = I_{disc(f)} + I_{child(f)}$$

$$= \frac{MR^2}{2} + m r_f^2$$

$$= \frac{MR^2}{2} + mR^2$$

[since child goes to edge (I disc is unchanged)]

$$= \left(\frac{M}{2} + m\right) R^2$$

$$\omega_i = 1 \text{ rev/s} = \frac{2\pi \text{ rad}}{1\text{s}} = 2\pi \text{ rad/s}$$

$$I_i \omega_i = I_f \omega_f$$

$$\Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$$

$$= \frac{\frac{MR^2}{2}(2\pi)}{\left(\frac{M}{2} + m\right) R^2}$$

$$= \frac{2\pi M}{M + 2m}$$

$$= \frac{2\pi (87)}{87 + 2(36)}$$

$$= \boxed{3.44 \text{ rad/s}}$$

Additional question: Is the kinetic energy conserved?

$$K_i = \frac{1}{2} I_i \omega_i^2$$

$$= \frac{1}{2} \frac{MR^2}{2} (2\pi)^2$$

$$= \boxed{4160 \text{ J}}$$

$$K_f = \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2} \left(\frac{M}{2} + m\right) R^2 \omega_f^2$$

$$= \boxed{2280 \text{ J}}$$

$\Rightarrow$ *Kinetic energy has decreased (Work done on the system, opposing the natural tendency of motion)*