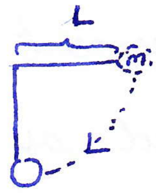
Work, Kinetic Energy, & Power

Limits of Newton's Second Law

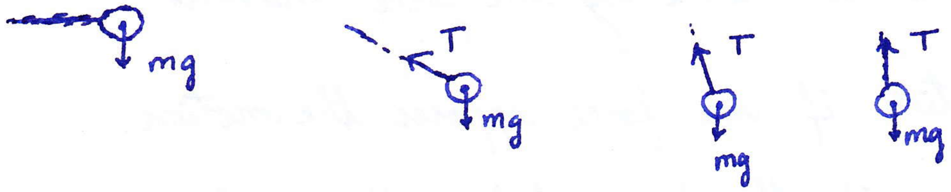
→ Not practical when acceleration is not constant.



Consider a pendulum released from a horizontal position, as shown in the figure to the right.

Question: What is velocity ~~at the to~~ of the bob at the lowest point in its trajectory?

If we approach this problem with Newton's Second Law, we run into difficulties. The free body diagrams for the pendulum's bob at different points during the bob's transit are shown below



The sum of the forces is not constant, so, from Newton's second law, the acceleration is not constant.

Examples: constant a: free fall, inclined plane

non-constant a: springs, pendulums, roller-coasters

<u>An alternate approach: Work-Energy methods.</u>

→ Allow us to relate velocities and positions directly, without first solving for times or accelerations. These are <u>often easier to use</u> than standard kinematics + dynamics.

Definition of work

If a constant force $\vec{F}$ acts on an object while it undergoes a displacement $\vec{d}$, then the work done ~~f~~ by that force is defined as

$$\boxed{W = F_{\parallel}\, d}$$

distance traveled

work done on object

component of force parallel to displacement

Work is a scalar quantity, with units of N.m = J (Joules)

~~W < 0 (negative) if~~

The sign of the work done by the force matters.

- W is negative if the force opposes the motion.
- W is positive if the force helps the motion.

Also, $W = 0$ if the force is perpendicular to the displacement ($\vec{F} \perp \vec{d}$)

★ Remember, this is a physicist's definition of work, and differs from the common motion of work that you might have. A force only does work if there is some motion parallel to the direction of the force. I could hold a bag full of bowling balls for a month, standing in one spot, but the physicist would still say that I had done no work!

Kinetic Energy

Kinetic energy is the energy associated with the motion of the object. An object of mass 'm' and velocity $\vec{v}$ has kinetic energy given by

$$\boxed{K = \frac{1}{2} m v^2}$$

The units of kinetic energy are $kg\left(\frac{m}{s}\right)^2 = kg\,\frac{m}{s^2}\,m = N \cdot m = J$

So, it has the same units as work.

Work-Energy Theorem

This is what will help us tackle problems with variable acceleration. The work-energy theorem states that:

* Net work done on an object equals the change in its kinetic energy.

$$\boxed{W_{Net} = \Delta K = K_f - K_i}$$

Example: Work-Energy Theorem applied to a block on an inclined plane.

→ A 3 kg box starts from rest and slides 2m down an inclined plane angled at 30° to the horizontal. The coefficient of kinetic friction between the block and the inclined plane is 0.1

• Our task: Find the work done on the block and see if it equals the change in kinetic energy.

To find the net work, we will find the values for work done by the individual forces on the ~~of~~ object and add them up.

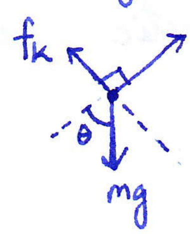
$$W_{net} = W_N + W_g + W_{f_K}$$

W_N: work done by normal force

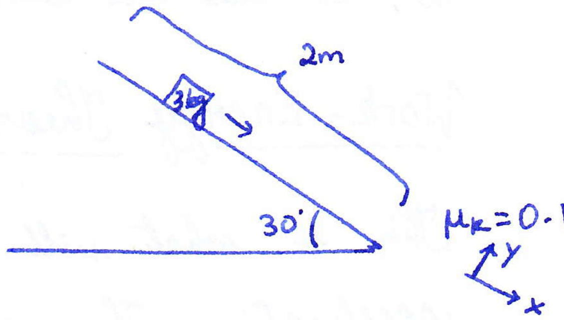
W_g: work done by ~~frictional~~ gravitational force

W_{f_K}: work done by frictional force.

Free-body-diagram



$W_N = 0$ since $\vec{N}$ is perpendicular to the displacement.



$$W_g = F_{g\parallel} d = mg\sin\theta\, d$$
$$= (3)(9.8)\sin 30^\circ (2)$$
$$= 29.4\text{ J}$$

$$W_{f_K} = -F_{f_K\parallel}\, d$$
$$= -(\mu_K)(N)\,d$$
$$= -(0.1)(mg\cos\theta)(2)$$
$$= -(0.1)(3)(9.8)\cos 30^\circ (2)$$
$$= -5.092\text{ J}$$

[The minus sign appears because the frictional force hinders the motion.]

$$\Rightarrow W_{net} = 0 + 29.4 - 5.092$$
$$= \boxed{24.3\text{ J}}$$

Now to check if this is the same as the change in the block's kinetic energy, ΔK

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}mv_f^2 - 0 = \frac{1}{2}mv_f^2$$

[$v_i = 0$, as the block starts from rest]

Applying Newton's Second Law ~~to~~ in the x-direction:

$$\sum_i F_{x\,i} = ma_x$$

$$\cancel{\Rightarrow -f_k + mg\cos\theta = ma}$$
$$\cancel{\Rightarrow -\mu_k N + mg\cos\theta = ma}$$
$$\cancel{\Rightarrow -\mu_k mg\cos}$$

$$\Rightarrow -f_k + mg\sin\theta = ma$$
$$\Rightarrow -\mu_k N + mg\sin\theta = ma$$
$$\Rightarrow -\mu_k mg\cos\theta + mg\sin\theta = ma$$
$$\Rightarrow -\mu_k g\cos\theta + g\sin\theta = a$$
$$\Rightarrow a = g(\sin\theta - \mu_k\cos\theta)$$

Since this acceleration is constant, we can apply the familiar kinematics equations.

$$v_f = v_0 + at$$
$$= 0 + g(\sin\theta - \mu_k\cos\theta)t$$

Now we need to find t.

$$x_f = x_i + v_0 t + \frac{1}{2}at^2$$
$$\Rightarrow 2\text{m} = 0 + 0(t) + \frac{1}{2}g(\sin\theta - \mu_k\cos\theta)t^2$$
$$\Rightarrow t = \sqrt{\frac{4}{g(\sin\theta - \mu_k\cos\theta)}} = 0.994\,\text{s}$$

$$\Rightarrow v_f = g(\sin\theta - \mu_k\cos\theta)t$$
$$= (9.8)(\sin 30^\circ - \overset{(0.1)}{\cancel{\mu_k}}\cos 30^\circ)(0.994)$$
$$= 4.026\ \text{m/s}$$

$$\Delta K = \frac{1}{2}mv_f^2 = \frac{1}{2}(3)(4.026)^2 = \boxed{24.3\ \text{J}}$$

Thus, we see that the net work done on the object equals the change in its kinetic energy.

□ Power: The rate at which work is done on an object, or the rate at which energy is transferred to the object.

Average power = $\boxed{\bar{P} = \frac{W}{\Delta t}}$ (analogous to average velocity)

Power has units of $\frac{J}{s} = W$ (Watt) (after the guy that invented the steam engine ~~3~~, James Watt.)

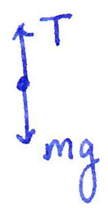
$$\bar{P} = \cancel{\frac{F_{\parallel}\Delta x}{\Delta t}}\ \frac{F_{\parallel} d}{\Delta t} = F_{\parallel}\frac{\Delta x}{\Delta t} = \cancel{F_{\parallel}\bar{V}}\ F_{\parallel}\bar{v}$$

$\Rightarrow$ Instantaneous power $= \boxed{F_{\parallel} v}$

Now let us look at a problem that incorporates all the concepts we have learned today.

- A 500 kg elevator initially at rest accelerates upwards at $0.5\ m/s^2$. Find the average power of the motor that runs the elevator in the first three seconds.

FBD of elevator.



Now, since we are ~~be~~ asked only for the power of the motor,

$$\bar{P}_{motor} = \frac{W_{motor}}{\Delta t}$$

$$W_{motor} = F_{motor\parallel}\, d = T d$$

The force of the motor is represented by the tension T in the rope pulling the elevator upwards.

From Newton's second Law: $\sum \vec{F} = m\vec{a}$

$$\Rightarrow T - mg = ma$$

$$\Rightarrow T = mg + ma$$

$$= m(g+a)$$

Now to find the distance ~~travelle~~ traveled by the elevator, d. We can use the kinematics equations since we have a constant ~~velocity~~ acceleration.

$$x_f = x_i + v_0 t + \frac{1}{2}at^2$$

$$\Rightarrow (x_f - x_i) = v_0 t + \frac{1}{2}at^2$$

$$\Rightarrow d = v_0 t + \frac{1}{2}at^2$$

$$\Rightarrow d = (0)(t) + \frac{1}{2}a(t)^2 = \frac{1}{2}(0.5)t^2$$ [$v_0 = 0$, since elevator initially at rest]

$$\Rightarrow d = \frac{t^2}{4} = \cancel{(3\ seconds)^2} \frac{3^2}{4} = \frac{9}{4}\ m$$

$$\Rightarrow W_{motor} = Td = m(g+a)d$$

$$= (500)(9.8+0.5)\left(\frac{9}{4}\right)$$

$$= 11587.5\ J$$

$$\Rightarrow P_{motor} = \frac{W_{motor}}{\Delta t} = \frac{11587.5}{3} = \boxed{3862.5\ W}$$